Two-dimensional nonlinear beam shaping

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We develop a technique for two-dimensional arbitrary wavefront shaping in quadratic nonlinear crystals by using binary nonlinear computer generated holograms. The method is based on transverse illumination of a binary modulated nonlinear photonic crystal, where the phase matching is partially satisfied through the nonlinear Raman–Nath process. We demonstrate the method experimentally showing a conversion of a fundamental Gaussian beam pump light into three Hermite–Gaussian and three Laguerre–Gaussian beams in the second harmonic. Two-dimensional binary nonlinear computer generated holograms open wide possibilities in the field of nonlinear beam shaping and mode conversion. © 2012 Optical Society of America

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Shaping a generated wavefront in nonlinear conversion adds functionality and opens exciting new possibilities such as all-optical self-routing and self-shaping of beams [1,2]. The alternative approach of shaping a beam in the fundamental frequency (FF) and then trying to frequency convert it usually does not work, except for some very simple cases [3], due to the difficulty of maintaining the phase-matching requirements with non-Gaussian beams. Only one-dimensional (1D) beam shaping in quadratic nonlinear crystals was ever experimentally demonstrated. Shaping the amplitude of the generated beam was suggested by means of varying the interaction length [4]. Shaping the phase was achieved by introducing a transverse-dependent phase term in a periodic structure [1,5] or in a nonlinear structure that generated multiple focal points at the converted frequency [6]. Shaping was also demonstrated in nonlinear wave mixing with non-Gaussian inputs [7]. In addition, it has been recently demonstrated that 1D arbitrary beam shaping is possible when introducing the concept of a continuous computer generated hologram (CGH) into the nonlinear optical regime [8]. Only 1D beam shaping was addressed due to the limitations of the fabrication technique of quadratic nonlinear crystals. The main method for modulating the nonlinear coefficient, electric field poling in ferroelectric crystals [9], is a planar method that enables use of only two of the three available axes of the nonlinear crystal.

A common method for two-dimensional (2D) beam shaping in linear optics is based on using CGH [10]. When a light beam is sent through a CGH, the far-field diffracted wavefront has the desired amplitude and phase. There are several ways of implementing the coding of information in a CGH—for example, encoding in a continuous [11] or a binary [12] form. Introducing a 1D continuous CGH into the nonlinear regime [8] was based on periodically modulating the propagation axis (the crystal's x axis) to enable quasi-phase matching, and only one additional axis (the crystal's y axis) was available for realization of the nonlinear holographic pattern. Due to the use of the two available axes in the crystal, it is impossible to achieve 2D beam shaping using this scheme.

In this Letter, we demonstrate for the first time, to our knowledge, arbitrary 2D beam shaping in binary nonlinear CGH. When a fundamental light beam passes through the crystal, a wavefront with the chosen amplitude and phase is obtained in the second harmonic (SH). This is achieved by combining a transverse setting of the crystal [13] with encoding of a 2D binary pattern. Unlike the 1D nonlinear CGH [8], here the crystal is set in a transverse setting [13]—i.e., the beam propagates along the crystal's z axis, and the two orthogonal axes are used for realizing the nonlinear holographic pattern. In addition, the FF is y polarized and not z polarized, as in the 1D scheme. Since both transverse axes are needed for beam shaping, how is it possible to maintain the phase-matching requirements? The solution we propose to this problem is to use the nonlinear Raman–Nath process [14], where only the transverse part of the vectorial phase-matching condition is fulfilled. In this configuration, we cannot use a coding function that relies on continuous modulation of the amplitude and phase, as is done in [8], since the only possibilities we have are either an entirely positive or an entirely negative quadratic susceptibility throughout the interaction region. Fortunately, such a case of binary modulation was already addressed by Lee [12], in the field of linear CGH. Lee suggested the following binary coding method for the amplitude transmittance function of a CGH:

$$t(x,y) = \begin{cases} 1 & \cos[2\pi f_{\text{carrier}} x - \varphi(x,y)] - \cos[\pi q(x,y)] \geq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where $q(x,y)$ is a function of the amplitude of the encoded information, defined by $\sin\,[\pi * q(x,y)] = A(x,y)$. $A(x,y)$ and $\varphi(x,y)$ are the amplitude and the phase, respectively, of the Fourier transform (FT) of the desired wavefront in the first diffraction order. $A(x,y)$ is normalized to the range 0–1, $\varphi(x,y)$ is in the range 0–2π, and $q(x,y)$ is in the range 0–0.5; f_{carrier} is the frequency of the carrier function. We can implement this function to realize a 2D nonlinear CGH. The 2D modulation of the second order nonlinearity coefficient of a crystal in this case is

$$d_{\text{NLO}}(x,y) = d_{ij}\,\text{sign}\{\cos[2\pi f_{\text{carrier}} x - \varphi(x,y)] - \cos[\pi q(x,y)]\}, \tag{2}$$

where d_{ij} is an element of the quadratic susceptibility $\chi^{(2)}$ tensor. Without the amplitude and phase modulation that are required for beam shaping, the nonlinear coefficient of the structure is periodically modulated with an angular frequency $2\pi f_{\text{carrier}}$. In this case, the nonlinear Raman–Nath process generates a SH beam at an angle $\theta_{\text{RN}} = \arcsin(2\pi f_{\text{carrier}}/k_{2\omega})$, which is equivalent to the angle of the first diffraction order for a regular grating with the same angular frequency [14]; this is schematically illustrated in Fig. 1(a). When the amplitude and phase modulation are added, the far-field SH beam profile at this first diffraction order is the FT of $A(x,y)\exp[i\phi(x,y)]$. It should be noted that the nonlinear Raman–Nath process does not provide full matching of the wave vectors, since the longitudinal part of the vectorial phase-matching condition is not fulfilled. We will discuss other phase-matching options in the last part of this Letter.

To demonstrate the concept of encoding a 2D CGH in nonlinear crystals we chose to fabricate a crystal aimed to generate three modes from the Hermite-Gaussian (HG) family [15], HG_{11}, HG_{12}, and HG_{21}, and three modes from the Laguerre-Gaussian (LG) family [15], LG_{01}, LG_{02}, and LG_{11}, in the first diffraction order. The transverse spatial distribution of HG modes at origin is

$$U_{l,m}(x,y,0) = A_{l,m} G_l\left[\frac{\sqrt{2}x}{W_0}\right] G_m\left[\frac{\sqrt{2}y}{W_0}\right] \quad (3)$$

where $A_{l,m}$ is the amplitude, W_0 is the waist, and $G_l(u)$ is the lth-order HG function. The transverse spatial distribution of LG modes at origin is

$$U_{l,p}(r,\phi,0) = \frac{C_{l,p}}{W_0}\left(\frac{\sqrt{2}r}{W_0}\right)^{|l|}\exp\left(-\frac{r^2}{W_0^2}\right)L_p^{|l|}\left[\frac{2r^2}{W_0^2}\right]\exp[il\phi], \quad (4)$$

where $C_{l,p}$ is the amplitude, $L_p^l(u)$ is a generalized Laguerre polynomial, and l is the topological charge of the beam. Since the FT of an HG mode and an LG mode is the same mode, the modulation of the nonlinearity coefficient was calculated according to Eqs. (3) and (4).

A schematic illustration of the proposed experimental setup is shown in Figs. 1(b) and 1(c), with simulations of the far-field images for one mode of each family, HG_{11} and LG_{11}. Numerical simulations were performed based on the split-step Fourier method. In addition, the illustration shows the poling pattern of the crystal computed for each of the two modes and the transverse setting in which the experiment took place.

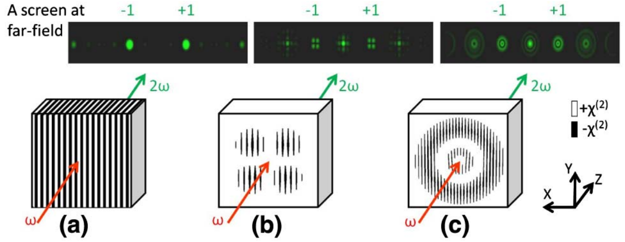

Fig. 1. (Color online) A Raman–Nath process in a periodically poled crystal (a) and the 2D binary nonlinear CGH setup and far-field results for HG_{11} (b) and LG_{11} (c).

We fabricated the suggested structure by 2D electric field poling of a stoichiometric lithium tantalate (SLT) nonlinear crystal. The crystal had six separate $0.5 \times 0.5\ \text{mm}^2$ structures, with different modulations of the second order nonlinearity coefficient, according to Eq. (2). Figure 2 presents a comparison between the calculated and the fabricated poling patterns for three of the structures. The microscope pictures are of one of the surfaces of the crystal after selective etching, and the good quality of the poling process can be observed. The thickness of the crystal was about 0.5 mm. The FF source used was a Nd:YAG laser producing 4.4 ns pulses at a 10 kHz repetition rate at a wavelength of 1064.5 nm. An ordinary polarized laser beam was focused to the center of the crystal, creating a waist radius of approximately 350 μm and an o-oo second harmonic generation was measured.

The desired HG modes were obtained at the first (left and right, +1 and −1) diffraction order. The frequency of modulation in the x direction, f_{carrier}, was chosen to be 0.035 μm^{-1}, hence the first diffraction order is obtained at an external angle of $\lambda_{\text{SH}} f_{\text{carrier}} \sim 18.6$ mrad, where λ_{SH} is the SH wavelength. Figure 3 presents a comparison between the theoretical and the measured profiles in the first diffraction order; spatial correlation between the measured and theoretical beam profiles is presented in Table 1. The simulated spatial correlation for all the beams in this Letter was higher than 0.95.

The desired LG modes were also obtained at the first diffraction order. The frequency of modulation in the x direction, f_{carrier}, was chosen in this case to be 0.06 μm^{-1}; the first diffraction order is obtained at an external angle of $\lambda_{\text{SH}} f_{\text{carrier}} \sim 31.9$ mrad. Figure 4 presents a comparison between the theoretical and the measured profiles in the first diffraction order, and the spatial correlation is presented in Table 1.

The LG_{11} mode is a vortex beam [16], carrying a topological charge of $l = +1$. The result shows that 2D nonlinear CGH is a method in which orbital angular momentum can be added through the nonlinear process, so that a nonvortex fundamental beam can be converted to a SH vortex beam. This type of conversion was theoretically discussed in [17] and recently experimentally demonstrated in spiral-shaped and fork-shaped nonlinear photonic crystals in [18]. In the latter crystals, only the generation of vortex beams is possible, whereas in this

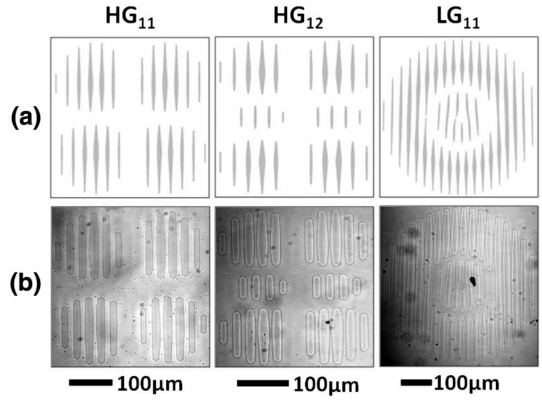

Fig. 2. A comparison between calculated (a) and fabricated (b) poling patterns in the HG_{11}, HG_{12} and LG_{11} structures.

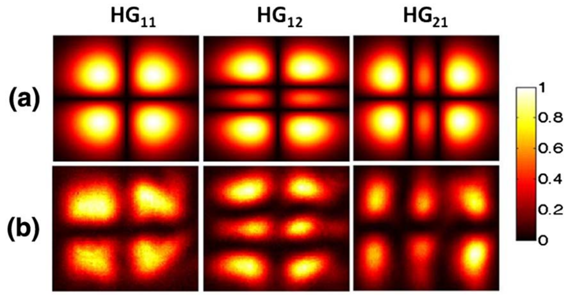

Fig. 3. (Color online) A comparison between theoretical (a), and measured (b) HG beam profiles at the first diffraction order.

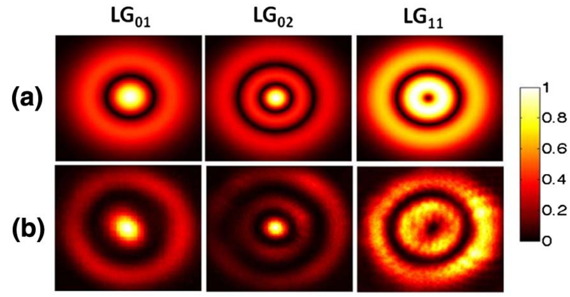

Fig. 4. (Color online) A comparison between theoretical (a), and measured (b) LG beam profiles at the first diffraction order.

Letter we present a general technique for generating arbitrary wavefronts.

We examined both the total SH generation and the power of the first diffraction order. The predicted external conversion efficiency, calculated for peak power, is $6.74 \times 10^{-12}\%$ W^{-1}, and the measured one is $11.2 \times 10^{-12}\%$ W^{-1}. A different fraction of the total SH power is diffracted to the first diffraction order in each structure; a comparison between the predicted and measured values is presented in Table 1. The fair agreement between the values indicates that the quality of modulation provided by the electric field poling technique is sufficient for realization of 2D nonlinear CGHs.

Two-dimensional binary nonlinear CGHs can be used to generate arbitrary shapes of beams and are not limited to only HG and LG beams. For example, they can be used to generate accelerating Airy beams [19,1], parabolic beams [20]. The limitation of the suggested scheme is that the full vectorial phase-matching condition is not fulfilled and hence the resulting conversion efficiency is quite small. An improvement of the efficiency can be achieved by working with the Cerenkov [21] phase-matching scheme, where only the longitudinal part of the vectorial phase-matching condition is fulfilled, or the Bragg [14] phase-matching scheme where the full vectorial condition is fulfilled. In these cases the required frequency of modulation, f_{carrier}, will be larger in comparison to the values mentioned here, which will require submicron patterning of the nonlinear crystal [22]. In the Bragg scheme, for our pump wavelength of 1064.5 nm, the required period is 0.98 μm. Conversion efficiency depends on the encoded information, e.g., for an Airy beam [19] in a 0.5 mm thick crystal, it is expected to be $7.7 \times 10^{-1}\%$ W^{-1}.

In conclusion, we have introduced a method for nonlinear 2D beam shaping, based on the concept of binary computer generated holograms. We experimentally demonstrated the method by converting a fundamental Gaussian beam pump light into three HG and LG beams at the second harmonic. Furthermore, nonlinear optical beam shaping can be employed for any desired wavefront and provides full control of the amplitude and phase of the converted beam. The ability to convert the frequency of light and to reshape its wavefront in any desired way is useful for all-optical shaping, routing, and switching of beams.

Table 1. Beam Profile Correlation and a Comparison between Predicted and Measured Percentage of SH in the First Diffraction Order

Mode	Spatial Correlation	Percentage of SH	
		Prediction	Measurement
HG_{11}	0.89	11.8%	8.1%
HG_{12}	0.9	9.12%	8.3%
HG_{21}	0.92	9.37%	9.5%
LG_{01}	0.81	27.7%	18.39%
LG_{02}	0.87	23.03%	20.57%
LG_{11}	0.92	26.9%	18.84%

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